

SUPPLEMENTARY MATERIAL A: THE ALGORITHM, STEP-BY-STEP

The algorithm learns a vocabulary of Roman-Numerals by observing a corpus of common-practice music and uses that vocabulary to tonally analyze some yet-unseen music.

The following therefore constructs an algorithm starts with no knowledge of Roman Numerals, triads, etc. and uses only the statistical properties of a musical corpus to undertake a key-finding task.

Overview: the steps involved to render raw statistics into a reduced vocabulary

- 1) Learning 1: Compile data & statistics from the raw musical surface of some corpus
- 2) Learning 2: Use the properties of these raw statistics to render a reduced Roman-Numeral-style harmonic vocabulary
- 3) Use the reduced vocabulary to analyze music

Materials: the Corpus

The Yale-Classical Archives Corpus consists of:

- 8,980 pieces by
- 505 composers with
- 12,415,229 salami-sliced (ever time a note is added or subtracted from the texture) verticalities totaling
- 3,818,712 quarter-notes of music
- Analyzed by key using music21's Bellman-Budge function (Cuthbert and Ariza 2011)
- For more information, reference White and Quinn (forthcoming) or visit ycac.yale.edu

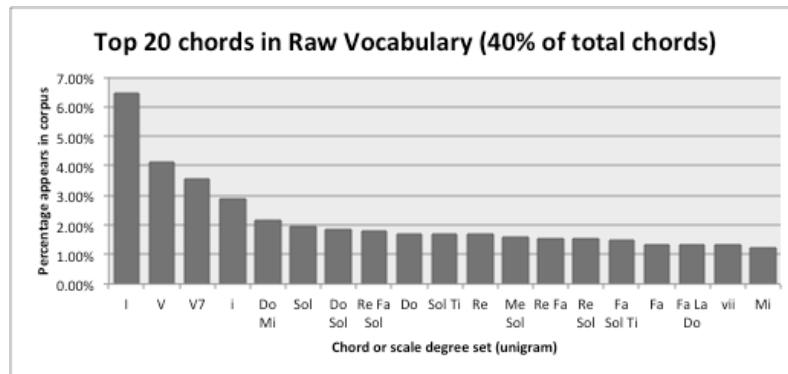
Learning 1: Compiling Raw Statistics

The learning process steps:

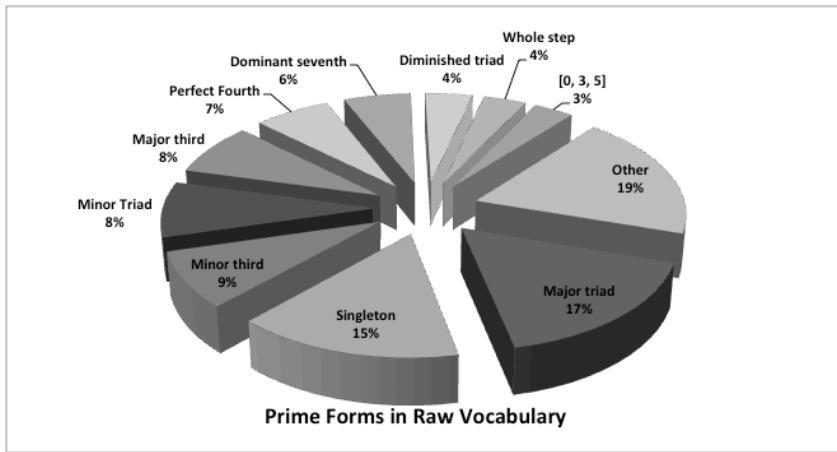
1. Remove repeats
2. If a set x of cardinality <3 is adjacent to its superset y , remove x .
3. Tally how often each verticality occurs to produce the **raw vocabulary**
4. Tally how often each succession occurs to produce the **raw transitions**

Raw Vocabulary:

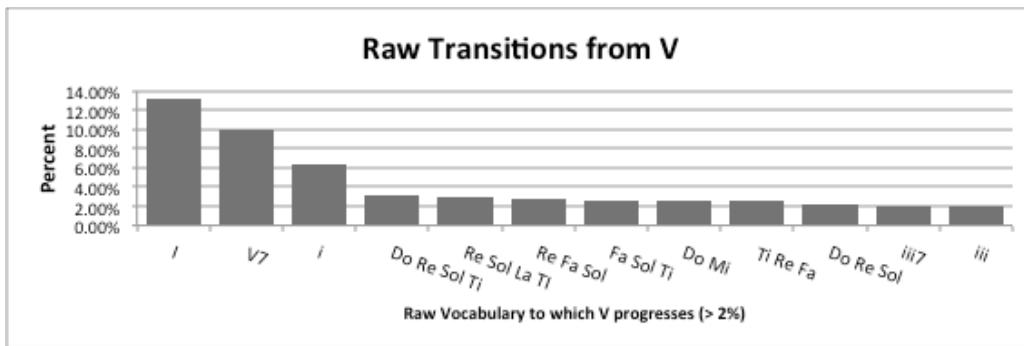
Below, the chart shows the 20 most frequent chords on the surface of the corpus. Note the differentiation of sets we would consider functionally equivalent, like V7 and $\langle\text{Sol Ti Fa}\rangle$, or I and $\langle\text{Do Mi}\rangle$.



The pie chart shows the prime forms present in the entire raw vocabulary. Note that the pie is very divided. For instance, all structures that appear $< 3\%$ are grouped into the “Other” slice, which occupies 19% of the distribution, the largest slice in the graph.



Below, the chart shows the chords to which V (or, <Sol, Ti, Re>) progresses on the raw surface. Again, note the differentiation of sets we would consider equivalent.



Learning 2: Manipulating the Raw Statistics to produce a Reduced Vocabulary

The Reduction Process steps:

- 1) Remove adjacent subsets
- 2) Remove chords not in top 85% of raw vocabulary (i.e., ignore improbable chords)
- 3) Remove singletons
- 4) Add or subtracts notes to increase the chords' contextual probability
 - Probability determined by raw transitions
 - Best solutions are determined by the Viterbi algorithm
- 5) Loop steps 1-4 till no further changes are made

The diagram illustrates a musical score with two staves (treble and bass) and a harmonic analysis grid below it. The grid shows chords as sets of notes (e.g., D, F, B, G). Four steps are highlighted:

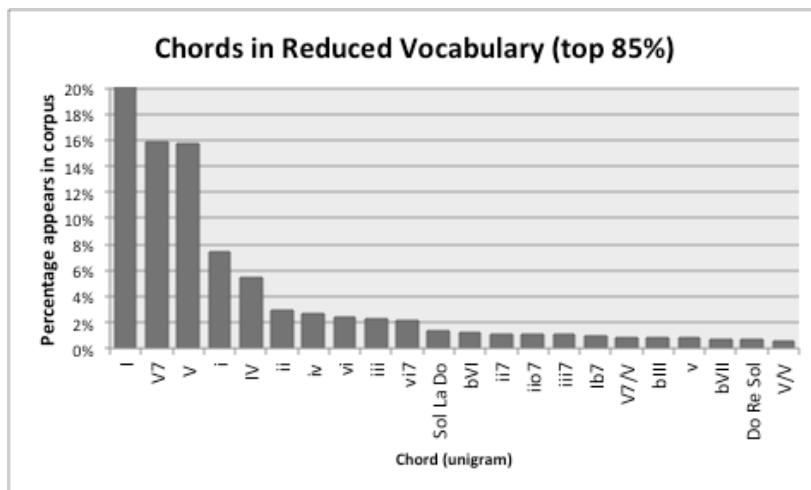
- Salami Slice**: Shows a vertical slice through the grid.
- Remove adjacent subsets < 2**: Shows the grid after removing small adjacent subsets.
- Add or subtract notes to increase contextual probability**: Shows the grid after adjusting notes based on context.
- Remove repeats**: Shows the final simplified grid.

Annotations explain specific decisions:

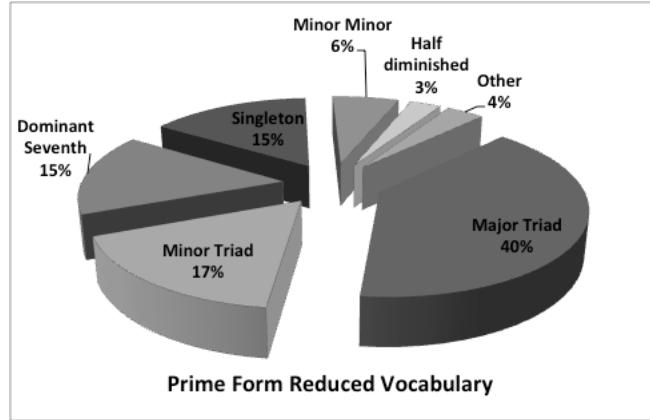
- A box states: "Removing this pitch will first create the far more frequent set, <G,B>. The next loop would then add D, creating the even-more-frequent <G,B,D>"
- A box states: "If the first three sixteenth notes are combined, removing this pitch would eliminate an improbable set <G,C#> between two identical and frequent <G,B,D> sets."

- 6) Tally how often each verticality occurs to produce the **reduced vocabulary**
- 7) Tally how often each succession occurs to produce the **reduced transitions**

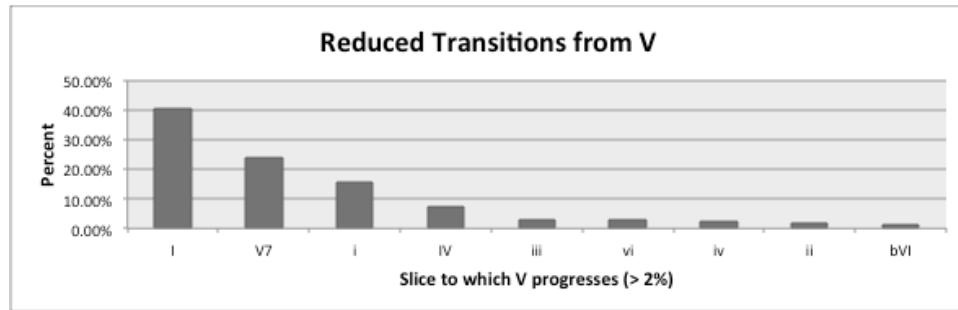
The chart below indicates the top 85% of chords in the new reduced vocabulary. Note the higher overall percentages of each vocabulary item compared to the raw vocabulary, indicating a more constrained distribution.



The pie chart indicates the prime forms within the reduced vocabulary. Note how triads and sevenths now dominate the distribution.



The graph below shows the frequencies of the chords from which V progresses in the reduced transitions. Note the more constrained distribution.



Mathematical Formalization of the Reduction Process

Formally, we can say that given a series of surface observations O with timepoints 1 to n such that...

$$O = (o_1, o_2 \dots o_n)$$

...we can reduce each o in O to a related chord s such that...

$$\begin{aligned} S &= (s_1, s_2 \dots s_n) \\ s_i \cap o_i &\geq 1 \end{aligned}$$

... where the intersection between each o and its corresponding s is at least 1 (i.e., they share at least one scale degree member). We produce the series S by maximizing the contextual probabilities k and the proximity of the two sets π .

$$S = \text{argmax } \prod_{i=1}^n P(s_i | k(o_i)) \pi(s_i, o_i)$$

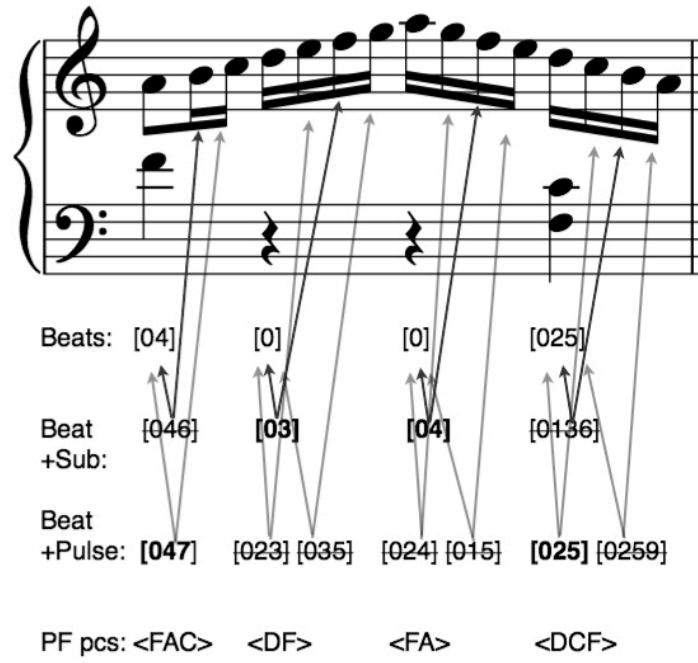
... where $P(s_i | k(o_i))$ is the probability that a given s would occur in the context in which we observe the corresponding o . In our case, this is a simple bigram probability (i.e., the

likelihood of seeing a chord given the chord that came right before it). Here, $\pi(s_i, o_i)$ is the proximity between the two sets, s and o . In our case, π is equal to the overlap (or intersection) between the two sets.

Analysis 1: Conform to Prime Forms of the Reduced Vocabulary

The steps:

1. Divide the music into three **metric levels**: the **beat**, **subdivision**, and **pulse**.
 - Zikanov 2014 is used to find recurrent periodicities in numbers of note onsets
2. List the prime forms that occur directly on each **beat**
3. For each beat, add the pcs of the next **subdivision**; list that prime form
4. For each beat & for each pulse surrounding the next subdivision: add the pcs of the next **pulses**; list those prime forms
5. If the process has found one or more prime forms present in the **vocabulary**, add the pitches of that prime form occurring most frequently in the vocabulary.



Otherwise, record only those pcs occurring on the **beat**

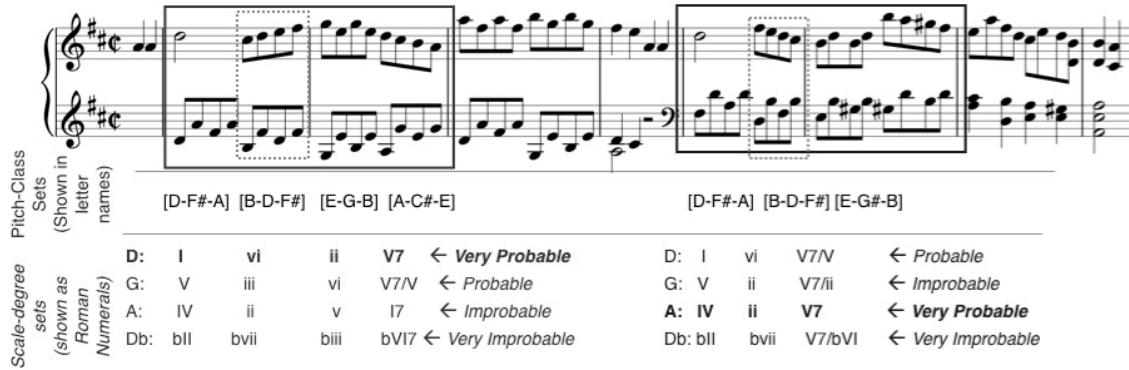
Analysis 2: Key Finding and Conforming to Reduced Vocabulary

Steps involved:

1. Select a window of music to analyze
2. For each possible key transposition of the window, proceed through the reduction process (see example in **Learning 2**):
 - a. Remove adjacent subsets
 - b. Removes chords not in top 85% of reduced vocabulary
 - c. Add or subtracts notes to increase the chords' contextual probability
 - d. Find best solution using Viterbi algorithm
3. Choose the key that assigns the window the most frequent (i.e., most probable) chord progressions

In its current incarnation, the program creates a window x beats long from each beat's time point. If the reduction process results in the number of chords falling below y , the following beat is added to the window. x and y are variable, with y representing the minimum length one expects the music to be in a single key.

Sample Analysis:



Things of note arising from discarded models:

- Reducing V7 to V (i.e., considering sevenths as subservient to triads in the vocabulary) creates models in which I-V and IV-I are often confused for one another
 - Highlights the importance the unique tritone of V7 in key-finding
- Not reducing to probable normal forms using meter often produces wildly inaccurate analyses, as the model confuses dissonances for chord tones
 - Highlights the connection between dissonance-consonance and improbable-probable events
 - Highlights the importance of triads and sevenths as structures separate from their scale-degree instantiations
- Not removing improbable chords from the vocabulary (i.e., taking 100% of chords rather than the 85% most probable, as reported above) creates analyses that are prone to error
 - Providing more vocabulary choices allows for more key ambiguity
- Admitting singletons into the vocabulary creates error-prone analyses
 - Highlights how ambiguous small-cardinality sets can be in the key-finding task

SUPPLEMENTARY MATERIAL B:
THE INSTRUCTION PAGE AND RESULTING HUMAN AND COMPUTER
ANALYSES OF THE KOSTKA-PAYNE EXCERPTS

INSTRUCTION PAGE

Name _____

Instrument _____

Age _____

Status: Freshman Sophomore Junior Senior Graduate Student

Gender Identity: Male Female GenderQueer or Neither Prefer not to respond

Analysis:

- Write Roman numerals under the music.
- Use the fewest possible RNs, approximately 1-2 per bar.
- Show longer prolongations by writing a single chord with a line following.
- For modulations, you write the numerals of the two different keys above one another.
- For chords you don't know, write a question mark ("?")
- Note sevenths ($V \neq V^7$).
- USE NO INVERSIONS (V^{43} or V^{65} will just be V^7).
- Do NOT analyze the greyed-out music, but you may take it into consideration when deciding on keys or modulations.

An example:

Allegretto

C: I V7----- I IV ? ii V I V -----

G: I ----- V7 -----

The following 32 analyses were created by sophomore music majors at the University of North Carolina at Greensboro's School of Music, Theater and Dance. The numbers below each example were randomly assigned to keep the identities of the students anonymous and to ensure their segregation from computer-generated examples.

Ex 1a, Anonymous (but often attributed to Bach), Minuet in G

7421

2a Bach, Chorale, "Uns ist ein Kindlein heut' geboren"

A handwritten musical score for a two-part chorale. The top part is in soprano C major, common time, and the bottom part is in bass F major, common time. The score consists of two staves. The soprano staff has a treble clef, a key signature of one sharp, and a common time signature. The bass staff has a bass clef, a key signature of one sharp, and a common time signature. Roman numerals are written below the notes to indicate harmonic progressions. The soprano starts with I, III, VII, I, I, IX, I. The bass starts with I, IV, vi, V, —, ii, ii, V, I. The score continues with a repeat sign and further harmonic analysis.

Handwritten harmonic analysis below the soprano staff:

- Measure 1: I
- Measure 2: III
- Measure 3: VII
- Measure 4: I
- Measure 5: I
- Measure 6: IX
- Measure 7: I

Handwritten harmonic analysis below the bass staff:

- Measure 1: I
- Measure 2: IV
- Measure 3: vi
- Measure 4: V
- Measure 5: —
- Measure 6: ii
- Measure 7: ii
- Measure 8: V
- Measure 9: I

4309

3a: Bach, "Jesu, der du meine Seele"

A handwritten musical score for Bach's "Jesu, der du meine Seele". The score consists of two staves of music in common time, with a key signature of one flat. The top staff starts with a forte dynamic. The bottom staff begins with a eighth-note bass line. Handwritten Roman numerals are placed below the notes to indicate harmonic progressions. In the first measure, the numerals are g:, i, VI, vii°, and 7. In the second measure, the numerals are II, IV, and F. In the third measure, the numerals are III, I, and II. The score continues with more measures, ending with a final measure where the numerals are I and II. The page number 533d is written in the bottom right corner.

533d

4a: Beethoven, Rondo op. 51, no. 1

103

ri - tar - dan

C: I

do pp

I

legato

cresc.

D^b: I II I V' I III

?

?

?

?

?

?

105

110

115 f

7976

5a: Beethoven, Sonata Op. 10 No. 1, ii

4913

6a: Beethoven, Piano Sonata op. 10, no. 3, ii

Dur: i V i V i IV IV IIIA

4025

7a: Beethoven, Sonata Op. 13, II

Adagio cantabile

p

I II⁷ III IV V VI VII |

9221

11a: Beethoven, String Quartet op. 135, iii

1082

12a: Beethoven String Trio op. 9, no. 3, ii

Adagio con espressione

p

p

p

I IV VI <>

pp *pp* — *rinf.* > *pp*

pp *pp* — *rinf.* > (*pp*)

II Viii bVI

2919

13a: Brahms, "Und gehst du ueber den Kirchhof", op. 44

espress.

doch lei - se fli - stern die Win - de, es

espress.

doch lei - se fli - stern die Win - de, es

espress.

doch lei - se fli - stern die Win - de, es

espress.

E I E I II IV V

hat - te zu heiss, zu heiss ge - liebt.

hat - te zu heiss, zu heiss ge - liebt.

I IV ? I B:VII vi I I

2773

14a: Ayer (arr. by Campbell), "Oh! You Beautiful Doll"

Handwritten musical score for "Oh! You Beautiful Doll" featuring three staves of music. The top staff shows measures 29 and 30, with lyrics "Oh, oh, oh!" and "Oh! you beau - ti - ful doll, — you great big beau - ti - ful doll! — Oh, oh, oh!". The middle staff shows measures 31 and 32, with lyrics "Let me put my arms a - bout you." and harmonic analysis markings: I, V, V7, V, I. The bottom staff shows measure 33, with lyrics "I could nev - er live with - out you. Oh! you" and harmonic analysis markings: I, V, V7, V, I. The score includes various musical markings such as slurs, grace notes, and dynamic signs.

8127

16a: Chopin, Mazurka op. 63, no. 2

Lento

p

fm Red. * Red. i III Red. ? VII Red. * Red. * Red. *

Red. * Red. * Red. * Red. * Red. * Red. *

vii^o Am II VII I

Red. * Red. * Red. * Red. * Red. *

?

10 15

5864

18a: Grieg, "The Mountain Maid," op. 67, no. 2

Allegretto tranquillo *p*

The musical score consists of three staves of music in common time, key signature of one flat. The top staff is for voice, the middle for piano (right hand), and the bottom for bassoon or cello. The vocal line begins with a melodic line of eighth and sixteenth notes. The piano accompaniment features sustained chords. The bassoon part provides harmonic support with sustained notes. The lyrics are:

She is slender and fair and young, With features so pure and pale; — The —
drooping lids of her eyes Their dreamy splen - dor veil. —

Harmonic analysis is indicated below the bassoon staff:

VI VI/V V V

Measure numbers 1, IV, 7, 1, ?, ?, ? are marked above the vocal line.

poco rit.

A large gray rectangular box covers the right side of the page, containing musical notation and text. The text includes:

(V) —
poco rit.
Reo.

Ex19a: Haydn, Sonata No. 22, iii

The image shows two staves of musical notation for piano, labeled "Finale" and "Tempo di Menuetto". The top staff is in common time (indicated by a '4') and the bottom staff is in 2/4 time. Both staves have treble clefs. The key signature is A major (three sharps). The music consists of six measures. Handwritten Roman numerals and Roman numerals with subscripts (I, II, III, IV, V, VI, VII) are written above the notes to indicate harmonic progressions. Measure 1 starts with I, followed by II, III, and IV. Measure 2 starts with IV, followed by VI, VII, and I. Measure 3 starts with VI, followed by V, I, and II. Measure 4 starts with I, followed by II, III, and IV. Measure 5 starts with IV, followed by V, VI, and VII. Measure 6 starts with VI, followed by V, I, and II.

2030

20a: Haydn, Piano Sonata no 30, I

A handwritten musical score for piano. The top staff shows two measures of music, labeled 84 and 85. Measure 84 consists of eighth-note chords in the treble clef. Measure 85 consists of eighth-note chords in the bass clef. Below the score, the key signature is indicated as F# major (F#:, one sharp) and the Roman numerals I⁽⁷⁾, VI⁷, IV⁷, and VII⁷ are written, corresponding to the chords in the measures.

0626

21a: Haydn, String Quartet op. 20, no. 4, i

13 15 20
tr
I — ? I — I ii — V⁷

5338

22a: Haydn, String Quartet op 50, no 6 ii

A handwritten musical score for a string quartet (two violins, viola, cello) in D major. The score consists of two systems of music. The top system starts with a dynamic of *fz* (fortissimo) and includes markings *m.v.*, *D.M.*, and Roman numerals I, IV, II, I, VI. The bottom system continues with Roman numerals I, II, and V. The score is annotated with various performance instructions such as *tr* (trill), *tr*, *tr*, *tr*, *tr*, and *tr*. The first system ends with a dynamic of *fz*.

1299

23a: Haydn, String Quartet op. 74, no 3, ii

A handwritten musical score for a string quartet (two violins, viola, and cello) in G major (two sharps). The score consists of four staves. Measure 30 starts with a single eighth note in the top staff. Measures 31-32 show rhythmic patterns of eighth and sixteenth notes. Measure 33 begins with a dynamic *p*. Measures 34-35 continue the rhythmic patterns. Handwritten lyrics in German are written below the staves: "G: I", "vi i", "ii", "v", "III". A large gray rectangular box covers the end of measure 35 through the beginning of measure 37.

9574

24a: Haydn, String Quartet op. 76, no. 6, ii

Handwritten musical score for Haydn's String Quartet op. 76, no. 6, ii, showing measures 31 and 35. The score is written on four staves (Violin I, Violin II, Viola, Cello) in common time. Measure 31 starts with a dynamic *p*. Measure 35 starts with a dynamic *pp*. Both measures feature various slurs and grace notes. Handwritten Roman numerals I, II, III, IV are placed below the staves to mark sections. The score is signed "1470" at the bottom right.

1470

25a: Mozart, Bassoon Concerto K. 191, ii

43

Ob.

Hn. in F

Bsn.

Vl. I

Vl. II

Vla.

Vc. D.B.

45

Ob.

Hn. in F

Bsn.

Vl. I

Vl. II

Vla.

Vc. D.B.

IV $\frac{V}{I}$ I $\frac{\#}{7}$ I

3.149

Ex 26a. Mozart, "Eine Kleine Nachtmusik", K. 525, ii

ROMANZE
Andante

I II I' II'

8325

27a: Mozart, Piano Concerto K. 488, ii

A handwritten musical score for the piano concerto. It consists of two staves: a treble staff and a bass staff. The key signature is F# major (one sharp). The tempo is marked 'Andante'. The score shows a melodic line in the treble staff with various note heads and stems, and harmonic support in the bass staff with bass notes and chords. Below the staves, there is handwritten text: 'f#: i in in II? ?'.

1650

28a: Mozart, Piano Sonata K. 330, ii

21
pp

i — iif iv ? i — V/ii iii ii

2720

30a: Mozart, Piano Trio K. 542, i

Handwritten musical score for Mozart's Piano Trio K. 542, first movement, page 30a. The score consists of four staves (treble, bass, two violins) and includes dynamic markings (p, f), tempo (210, 215), and harmonic analysis (VI, IV, V, VII). The score is written in common time, with a key signature of three sharps.

210

p

VI

IV

215

f

f

e.

VI

VII

41972

33a: Schubert "Erlkonig", mm. 113-23

113

115

"Ich

lie - be dich, mich reizt dei - ne schö - no Ge - stalt; und

pp

b II *ii⁰⁷* *b III*

120

bist du nicht wil - lig, so brauch ich Ge - walt."

vii⁰⁷ *vii⁰⁷/V* *II* *I*

7035

34a: Schubert, "Erlkonig", mm. 134-48

Musical score for Schubert's "Erlkonig" (mm. 134-48). The score consists of two staves. The top staff is for the voice (soprano) and the piano right hand. The bottom staff is for the piano left hand. The lyrics are written below the notes.

Measure 134: grau - set's, er rei - tet ge - schwind, er hält in
9: cresc.

Measure 135: Amen das lich - zen - de Kind.
ff

Measure 140: er - reicht den Hof mit Müh und
fz fz

Handwritten markings:
 - Measure 134: A circled '9' is written above the first measure.
 - Measure 135: 'cresc.' is written below the vocal line.
 - Measure 140: 'ff' is written below the vocal line.
 - Measure 140: 'fz fz' is written below the piano line.
 - Measure 140: Roman numerals 'i', '?', 'i', and 'iv' are written below the piano line.

8516

37a: Schubert, String Trio D. 471

187

pp

cresc.

pp

cresc.

pp

I VI I VI I ? cresc.

190

191

192

193

194

195 f

f

f

?

I 231

38a: Schubert, Originaltaenze, op. 9, no. 14.

A handwritten musical score for piano, featuring four staves of music. The score is in 3/4 time, with a key signature of four flats. Measure numbers 1 through 20 are indicated above the staves. The music consists of two systems. The first system ends at measure 15, with a fermata over the right hand's notes. The second system begins at measure 16, with a dynamic marking of *mf*. The score includes various markings such as *p*, *d.*, *10*, *15*, *20*, and *A:*. Handwritten Roman numerals I, II, III, IV, V, and VI are placed above the staves to indicate harmonic progressions. A large handwritten number 1 is positioned below the first staff of the second system.

4762

41a: Schumann, "Aus meinen Thraenen spriessen," op. 48, no. 2

Nicht schnell

p

Aus mei - nen Thränen spriesen viel bli - hen-de Blu-men her - vor,
und mei - ne Seuf - zer

p

10

wer - den ein Nach - ti-gal - len - chor. Und wenn du mich lieb hast, Kind - chen, schenk'ich dir die Blu-men

pp *p*

ritard.

all', und vor dei - nem Fen - ster soll klin - gen das Lied der Nach - ti - gall.

pp

ritard.

pp

III VI^{1/2} III VI^{1/2} VII VI^{1/2} — IV I

8530

42a: Schumann, Tragoedie, op. 136, no. 3

Rasch und mit Feuer *f sf*

Ent - flich' mit mir und sei mein Weib und ruh' an

s.f. *f*

I II III

Rwd. * Rwd. Rwd. Rwd.

mei - nem Her - zen aus! In wei - ter Fer - ne sei mein

IV V VI

Rwd. Rwd. Rwd. *

Herz dir Va - ter - land und Va - - ter - haus!

s.f. *sf*

V V V

Rwd. *

0656

43a: Schumann, "Wenn ich in deine Augen seh"

Langsam

p

Wenn ich in dei . ne Au - gen seh, so

G. I

schwin . det all mein Leid und Weh; doch wenn ich küss - se dei . nen

F: V7

Mund, so werd' ich ganz und gar ge - sund. Wenn

Ic

ich mich lehn' an dei . ne Brust, komm's ü . ber mich wie Himmels.

V

6

6053

44a: Tchaikovsky, "Morning Prayer," op. 39, no. 1

Lento

The musical score consists of three staves of music in 3/4 time with a key signature of one sharp. The first staff shows a progression from G major to D major. The second staff begins with a forte dynamic (f) and shows a progression through C major. The third staff concludes with a repeat sign (oda).

Handwritten harmonic analysis below the score:

- G:** I II III V⁷ (II)
- D:** V⁷/ii II I (IV)
- C:** II I V

7082

The following 32 analyses were created by the reduced YCAC model, with parameters set as described in the above study.

Ex 1a, Anonymous (but often attributed to Bach), Minuet in G

A handwritten musical score for a minuet in G major, 3/4 time. The score consists of two staves. The top staff shows melodic lines with various note heads and rests. The bottom staff shows harmonic bass lines. Handwritten Roman numerals are placed above the top staff and below the bottom staff, indicating harmonic progressions. A section of the score from measure 11 to measure 17 is highlighted with a grey rectangular box.

Handwritten Roman numerals above the top staff:

- I
- II
- I
- V⁺
- I

Handwritten Roman numerals below the bottom staff:

- III - I
- V

8010

2a Bach, Chorale, "Uns ist ein Kindlein heut' geboren"

Handwritten musical score for two staves. The top staff is in G major (G-C) and the bottom staff is in D major (D-II-V-I). Various Roman numerals are written above the notes, such as I, VI, II, VII, V, and II. The score includes measures 1 through 10.

8214

3a: Bach, "Jesu, der du meine Seele"

Handwritten musical score for Bach's "Jesu, der du meine Seele". The score consists of two staves in common time, key signature of one flat. The vocal part has lyrics in German and Roman numerals below them. The piano accompaniment is shown in the bass staff.

Handwritten harmonic analysis below the score:

- Measures 1-4: $\text{g-} \text{i} \text{ II} \text{i} \text{ V} - \text{ii}^7$
- Measure 5: V^7
- Measures 6-9: $\text{B}^5 \text{ II} \text{ I} - \text{ii}^7$
- Measure 10: II
- Measures 11-12: $\text{F: V} \text{ II} \text{ I} - \text{I}$

4685

4a: Beethoven, Rondo op. 51, no. 1

Handwritten musical score for Beethoven's Rondo op. 51, no. 1. The score consists of four staves of music, each with a key signature of one sharp (F#) and a time signature of common time (indicated by a 'C'). The music begins at measure 103.

- Staff 1:** Measures 103-105. The vocal line features eighth-note patterns. The lyrics "ri - tar - dan -" are written above the staff. Measure 105 ends with a fermata over the bassoon part.
- Staff 2:** Measures 106-110. The piano part is marked "do pp". The bassoon part is marked "cresc.". Measures 110-115 show a transition, indicated by question marks and Roman numerals I, II⁺, and I.
- Staff 3:** Measures 116-121. The piano part is marked "f". The bassoon part is marked "cresc.". Measures 121-126 show a continuation, indicated by question marks and Roman numerals I, II⁺, and I.

Handwritten annotations include:

- Roman numerals I, II⁺, and I placed above the staff in measures 106-115 and 116-121.
- A question mark "?" placed below the staff in measures 106-115 and 116-121.
- The word "legato" placed above the piano part in measure 110.
- The word "cresc." placed above the bassoon part in measure 110.
- A dynamic marking "f" placed above the piano part in measure 116.

9744

5a: Beethoven, Sonata Op. 10 No. 1, ii

Musical score for Beethoven's Sonata Op. 10 No. 1, ii, Adagio molto section. The score is in 2/4 time, key signature is B-flat major (two flats). The piano part shows various chords and fingerings (e.g., 3-4, 5-4, 8-7, 5-4, 8-7) and dynamics (e.g., p, cresc., fp). A circled '5' is above the measure starting at bar 32. Below the score, a handwritten harmonic analysis indicates the progression: A: I V⁷ I V⁷ I V.

6487

6a: Beethoven, Piano Sonata op. 10, no. 3, ii

The image shows two staves of handwritten musical notation for piano. The top staff is in treble clef and the bottom staff is in bass clef. Measure numbers 9 and 10 are indicated above the staves. The music consists of eighth-note patterns with various dynamics like *r*, *f*, and *mf*. Handwritten Roman numerals below the staves indicate harmonic progressions: *i*, *IV*, *i*, *IV* in the first section, and *C*, *iv*, *IV*, *I* in the second section. A shaded gray area covers the end of the second section. In the bottom right corner, there is a handwritten number *4855*.

7a: Beethoven, Sonata Op. 13, II

Adagio cantabile

p

I II V² II II+ VI
E V I

1025

11a: Beethoven, String Quartet op. 135, iii

6898

12a: Beethoven String Trio op. 9, no. 3, ii

Adagio con espressione

1 2 3 4 5

I *II* *IV* *V* *vi*

G *D* *I* *IV* *V*

C *II* *i* ? *Az6* *V*

4316

13a: Brahms, "Und gehst du ueber den Kirchhof", op. 44

The image shows a handwritten musical score for Brahms' "Und gehst du ueber den Kirchhof", op. 44. The score consists of four staves of music in G major (two sharps) and common time. The vocal line is in soprano C-clef, and the piano accompaniment is in bass F-clef. The vocal line is marked "espress." throughout. The lyrics are:

doch fei - se flü - stern die Win - de, es
 doch fei - se flü - stern die Win - de, es
 doch fei - se flü - stern die Win - de, es
 hat - te zu heiss, zu heiss ge - liebt.

Below the music, handwritten Roman numerals indicate the harmonic progression:

E: I vi \overline{IV} \overline{V}^7

\overline{I} \overline{II} \overline{III} \overline{VI} \overline{III} \overline{V}^7 \overline{I}

5050

14a: Ayer (arr. by Campbell), "Oh! You Beautiful Doll"

Handwritten musical score for "Oh! You Beautiful Doll" featuring three staves of music. The top staff shows a vocal line with lyrics and harmonic analysis below the notes. The middle staff continues the vocal line with harmonic analysis. The bottom staff concludes the section with harmonic analysis. The score includes lyrics such as "Oh, oh, oh!", "you beau - ti - ful doll, — you great big beau - ti - ful doll! —", "Let me put my arms a - bout you.", "I could nev - er live with - out you. Oh! ... you", and "I could nev - er". The harmonic analysis uses Roman numerals with subscripts and superscripts (e.g., I^b, II^b, III^b, IV^b, V^b, VI^b) and other symbols like A^b, C^b, and D^b.

T —

6465

16a: Chopin, Mazurka op. 63, no. 2

5068

18a: Grieg, "The Mountain Maid," op. 67, no. 2

Allegretto tranquillo *p*

The musical score consists of four staves of music in common time, key signature of one flat. The vocal line is in soprano C-clef, and the piano accompaniment is in bass F-clef. The vocal part begins with a melodic line, followed by piano chords. Handwritten Roman numerals indicate harmonic progressions: (IV) I, bII bVII, II, IV+, and V+. A 'poco rit.' marking is placed between the first two harmonic changes. The vocal line continues with lyrics: 'She is slender and fair and young, With features so pure and pale; The drooping lids of her eyes Their dreamy splen - dor veil.' The piano accompaniment includes a dynamic 'p' and a 'poco rit.' marking. A 'Rea.' (reprise) instruction is written near the end of the piano part. Below the score, handwritten Roman numerals show the progression: F: IV bII bVII, D: I II+, C: II IV+ II.

2982

Ex19a: Haydn, Sonata No. 22, iii

Finale
Tempo di Menuetto

E: I — III I IV I

II V I V I

5577

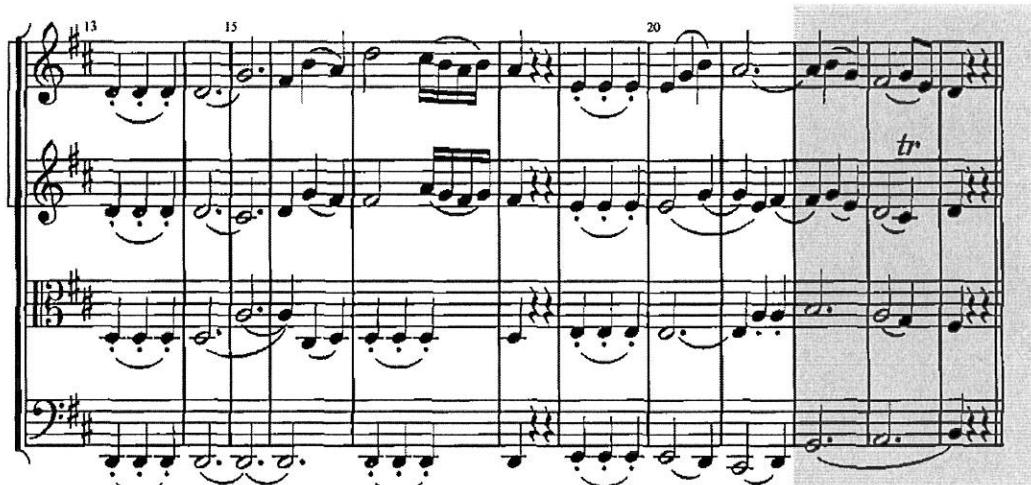
20a: Haydn, Piano Sonata no 30, I



F^{\sharp} I^+ VI
 $A: \text{vi}$ $\text{III} \text{IV}^+$

3076

21a: Haydn, String Quartet op. 20, no. 4, i



I (2) I V⁷ (1) V⁷ I

4416

22a: Haydn, String Quartet op 50, no 6 ii

A handwritten musical score for a string quartet. The score consists of four staves: Violin I (top), Violin II, Viola, and Cello (bottom). The key signature is one sharp, and the time signature is common time. Measure 55 is indicated at the top left. The score features dynamic markings such as *fz* (fortissimo) and *m.v.* (mezzo-vivo). The violins play eighth-note patterns, while the viola and cello provide harmonic support. Handwritten Roman numerals I, II, III, and IV are placed above the staves to mark sections. The score is divided into two main sections by a vertical grey bar, with section II starting after the bar. Below the score, the Roman numerals I, II, III, and IV are written again.

2628

23a: Haydn, String Quartet op. 74, no 3, ii

Handwritten musical score for Haydn's String Quartet op. 74, no 3, ii, showing measures 30-35. The score is in common time, key signature of one sharp, and consists of four staves: Violin I, Violin II, Viola, and Cello. Measure 30 starts with a single note in Violin I followed by a rest. Measure 31 begins with a sustained note in Violin II. Measures 32-33 show rhythmic patterns in the strings. Measure 34 features a bassoon-like line in the cello. Measure 35 concludes with a sustained note in the violins. Handwritten Roman numerals below the staff indicate harmonic progressions: I, II, III, IV, V, VI, VII, and II. The bassoon line in measure 34 is labeled with 'e:'. Measure numbers 30 and 35 are written above the staff.

4405

24a: Haydn, String Quartet op. 76, no. 6, ii

31

B-I II- I- II- III-

35

i- II- i- ?

B- IV- vii- II- +

25a: Mozart, Bassoon Concerto K. 191, ii

Ob.

Hn. in F

Bsn.

Vl. I

Vl. II

Vla.

Vc. D.B.

Ob.

Hn. in F

Bsn.

Vl. I

Vl. II

Vla.

Vc. D.B.

43

44

45

I — III — I
II — IV⁷

tr

3

? IV⁷/II — I

6085

Ex 26a. Mozart, "Eine Kleine Nachtmusik", K. 525, ii

ROMANZE
Andante

p *f*
p *f*
f
f

p *f*
C **I** **ii** **IV** **iii** **VII** **I** —

p
p
p
p

ii **V** **I**

27a: Mozart, Piano Concerto K. 488, ii

Andante

F# i ~~IV~~ . . . III⁷ i II⁷ i I ~~IV~~

A: ~~IV~~ VI⁷

6109

28a: Mozart, Piano Sonata K. 330, ii

A handwritten musical score for a piano sonata. The score consists of two staves: treble and bass. The key signature is three flats. Measure 21 begins with a dynamic of *pp*. The melody is primarily in the treble staff, while the bass staff provides harmonic support with sustained notes and eighth-note patterns. Handwritten markings include a smiley face above the first measure, a question mark over a bass note in the second measure, and a large question mark with a checkmark over the bass note in the third measure. The score ends with a fermata over the final note of the bass staff.

6980

30a: Mozart, Piano Trio K. 542, i

210

A: V⁺ II IV I E II II

C: V/vi

215

f

f

C: I ii I IV

7915

33a: Schubert "Erlkonig", mm. 113-23

113

115

"Ich

E♭ ↓

3 3

pp

I

II

III

fff

d: vii ot

120

lie - be dich, mich reizt dei - ne schö - ne Ge - stalt; und

bist du nicht wil - lig, so brauch ich Ge - walt."

6642

34a: Schubert, "Erlkonig", mm. 134-48

134 135

grau - set's, er rei - tet ge - schwind, er hält in

cresc.

C V 3

Ar - men das äch - zen - de Kind.

ff

er - reicht den Hof mit Müh und

fz fz

— I

A^b iii ————— I

7299

37a: Schubert, String Trio D. 471

187

pp

cresc.

pp

cresc.

pp

cresc.

pp

cresc.

195

fp

f

fp

f

fp

f

A: *V* *III*

V *I*

1454

38a: Schubert, Originaltaenze, op. 9, no. 14.

A handwritten musical score for piano, featuring four staves of music. The score is in common time and consists of four measures per system. The key signature changes frequently, indicated by Roman numerals above the staff. Measure 1 starts in A minor (I), measure 2 in D major (IV), measure 3 in A major (V), and measure 4 in E major (VI). The dynamics are marked with 'p' (piano) and 'mf' (mezzo-forte). The score includes various note heads, rests, and bar lines. The first system ends at measure 4, and the second system begins at measure 5. The handwritten text 'I' is written below the bass staff of the second system.

0080

41a: Schumann, "Aus meinen Thraenen spriessen," op. 48, no. 2

Nicht schnell

p

Aus mein-en Thränen spries-sen viel blü-hen-de Blu-men her - vor, und mein-e Seuf-zer

p

A: I — II — IV — V — VI —

wer-den ein Nach - ti - gal - len - chor. Und wenn du mich lieb hast, Kind - chen, schenk'ich dir die Blu-men

pp *p*

VII — I — II — V — VI — ritard.

all', und vor dei - nem Fen - ster soll klin - gen das Lied der Nach - ti - gall.

pp

Re. *

ritard.

pp

D: I VII — IV — II —

5566

42a: Schumann, Tragoedie, op. 136, no. 3

Rasch und mit Feuer *f sf*

Ent - flich' mit mir und sei mein Weib und ruh' an

s.f. *f*

Ped. * Ped. Ped. Ped. *v* *v* *v* *v*

mei - nem Her - zen aus! In wei - ter Fer - ne sei mein

v *v* *v* *v*

Ped. *V*⁷ Ped. *IV*⁷ Ped. *III*⁷ Ped. *II*⁷ Ped. *I*⁷ Ped. *V*⁷ *

Herz dir Va - ter - land und Va - - ter - haus!

s.f. *s.f.* *Ped.* *

q742

43a: Schumann, "Wenn ich in deine Augen seh"

Langsam

p

Wenn ich in dei - ne Au - gen seh', so

G: I

schwin - det all mein Leid und Weh; doch wenn ich küss - se dei - nen

vi ii I F: iii V⁷

Mund, so werd' ich ganz und gar ge - sund. Wenn

I V c: VI iv

ich mich lehn' an dei - ne Brust, kommt's ü - ber mich wie Himmels.

c: V i IV V⁷ 8099

44a: Tchaikovsky, "Morning Prayer," op. 39, no. 1

Lento

p

mf

f

I II III IV V VI VII VIII IX X XI XII XIII XIV XV

I II III IV V VI VII VIII IX X XI XII XIII XIV XV

VII VI V IV V I II III IV V VI VII VIII IX X XI XII XIII XIV XV

2485